- **5340:** *Proposed by Oleh Faynshteyn, Leipzig, Germany*

Let a, b and c be the side-lengths, and s the semi-perimeter of a triangle. Show that

$$\frac{a^2+b^2}{(s-c)^2}+\frac{b^2+c^2}{(s-a)^2}+\frac{c^2+a^2}{(s-b)^2}\geq 24.$$

Solution 3 by Arkady Alt, San Jose, CA

Note that $\sum_{cyc}\frac{a^2+b^2}{(s-c)^2}\geq 24 \iff \sum_{cyc}\frac{a^2+b^2}{(a+b-c)^2}\geq 6.$

Since $a^2\geq a^2-(b-c)^2 \iff \dfrac{a^2}{a+b-c}\geq c+a-b$

and

$$b^2\geq b^2-(c-a)^2 \iff \frac{b^2}{a+b-c}\geq b+c-a$$

then by AM-GM Inequality we have

$$\sum_{cyc}\frac{a^2}{(a+b-c)^2}\geq\sum_{cyc}\frac{c+a-b}{a+b-c}\geq 3\sqrt[3]{\frac{c+a-b}{a+b-c}\cdot\frac{a+b-c}{b+c-a}\cdot\frac{b+c-a}{c+a-b}}=3$$

and

$$\sum_{cyc}\frac{b^2}{(a+b-c)^2}\geq\sum_{cyc}\frac{b+c-a}{a+b-c}\geq 3\sqrt[3]{\frac{b+c-a}{a+b-c}\cdot\frac{c+a-b}{b+c-a}\cdot\frac{a+b-c}{c+a-b}}=3.$$

Thus, $\sum_{cyc}\frac{a^2+b^2}{(a+b-c)^2}\geq 6.$